

Problem Set 3

Please complete and submit **any ONE** of the following problems. The deadline for submission is January 7 by 11:59 PM. Send your submission by email to benjamin.fedoruk@ontariotechu.ca with the header "[Surname] Problem 3".

1. Let z and i be the identities for addition and multiplication in some field K . Prove that z and i are unique – that is, show that there is only one element that functions like z and i .
2. Let z be the identity for addition in some field K . Prove that for any $k \in K$, $z \times k = z$.
3. Let i be the identity for multiplication in some field K . Prove that for any $k \in K$, $(-i) * k = -k$ where $-i$ is the additive inverse of i and $-k$ is the additive inverse of k .
4. Let K be a field and let $K(\sqrt{x_1}, \sqrt{x_2}, \dots, \sqrt{x_d})$ denote the set $\{k_0 + k_1\sqrt{x_1} + k_2\sqrt{x_2} + \dots + k_d\sqrt{x_d} \mid k_1, k_2, \dots, k_d \in K\}$. Prove that $K(\sqrt{x_1}, \sqrt{x_2}, \dots, \sqrt{x_d})$ is a field. (Hint: Use induction.)
5. True or false: Let K be a field. For any $k_1, k_2 \in K$, if $k_1 \times k_2 = 0$ then either $k_1 = 0$ or $k_2 = 0$. Justify your answer with either a proof (if true) or a counterexample (if false).