Tutorial 10

MATH3020: Real Analysis
9 - Continuity in $\mathbb{R}$

Benjamin Fedoruk

Ontario Tech University

November 19, 2024

Question 1 - A Proof on Continuity

(Wade 3.3.5/91) If f is a real function which is continuous at $a \in \mathbb{R}$ and if $f(a) < M$ for some $M \in \mathbb{R}$, prove that there is an open interval I containing a such that $f(x) < M$ for all $x \in I$.

(Wade 3.3.3/91) If $f : [a, b] \to \mathbb{R}$ is continuous, prove that $\sup_{x \in [a,b]} |f(x)|$ is finite.

Question 3 - Continuity and Fixed Points

(Wade 3.3.4/91) If $f : [a, b] \to [a, b]$ is continuous, then f has a fixed point; that is, there is a $c \in [a, b]$ such that $f(c) = c$.

(Wade 3.4.1/96) Using the definition, prove that each of the following functions is uniformly continuous on $(0, 1)$:
$f(x) = x^2 + x, g(x) = x^3 - x + 2, h(x) = x \sin 2x$.