Tutorial 9

MATH3020: Real Analysis

Extra - Double Sequences and Their Limits

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Question 1 - Constant Row/Column Theorem

Let $(a_{n,m})$ be a double sequence. If for some $k \in \mathbb{N}$, $a_{i,j} = a_{i,k}$ for all $i \in \mathbb{N}$ and $j \geq k$ then $\lim_{m\to\infty} a_{n,m}$ exists for each n, and particularly is equal to $a_{n,k}$.

Question 2 - Zero Product Theorem

Let $(a_{n,m})$ be a double sequence where $a_{n,m} = b_n c_m$ for sequences (b_n) and (c_m). If $\lim_{n\to\infty} b_n = 0$ and (c_m) is bounded, then $\lim_{n\to\infty}(\lim_{m\to\infty} a_{n,m}) = 0$.

Question 3 - Diagonal Sequence

Let $(a_{n,m})$ be a double sequence and define $d_n = a_{n,n}$. Prove or disprove: If $\lim_{n\to\infty} d_n$ exists, then $\lim_{(n,m)\to\infty} a_{n,m}$ exists.

Question 4 - MCT for Double Sequences

Prove the Monotone Convergence Theorem for double sequences.