

Tutorial 7

MATH3020: Real Analysis

7 - Limit Theorems

8 - Bolzano-Weierstrass Theorem

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Question 1 - Archimedes' Pi Approximation

(Wade 2.3.10/58) Suppose that $x_0 = 2\sqrt{3}$, $y_0 = 3$. For $n \in \mathbb{N}$, let

$$x_n = \frac{2x_{n-1}y_{n-1}}{x_{n-1} + y_{n-1}}$$

and

$$y_n = \sqrt{x_n y_{n-1}}$$

- 1 Prove that $x_n \downarrow x$ and $y_n \uparrow y$ as $n \rightarrow \infty$ for some $x, y \in \mathbb{R}$.
- 2 Prove that $x = y$ and $3.14155 < x < 3.14161$. (Note: The actual value of x is π .)

Question 2 - Construction of Sequence Converging to Supremum

(Wade 2.3.7/57) Suppose that $E \subset \mathbb{R}$ is a nonempty bounded set and that $\sup E \notin E$. Prove that there exists a strictly increasing sequence $(x_n) \in E^{\mathbb{N}}$ that converges to $\sup E$.

Question 3 - Rational Approximation of Reals

(Wade 2.2.6) Prove that given $x \in \mathbb{R}$ there is a sequence $r_n \in \mathbb{Q}$ such that $r_n \rightarrow x$ as $n \rightarrow \infty$.