

# Tutorial 5

MATH3020: Real Analysis

## 6 - Sequences in $\mathbb{R}$

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# Question 1 - Convergence Properties

(Wade 2.1.0/45) Determine which of the following statements are true and which are false. Prove the true ones and provide a counterexample for the false ones.

- 1 If  $x_n$  converges, then  $\frac{x_n}{n}$  also converges.
- 2 If  $x_n$  does not converge, then  $\frac{x_n}{n}$  does not converge.
- 3 If  $x_n$  converges and  $y_n$  is bounded then  $x_n y_n$  converges.
- 4 If  $x_n$  converges to zero and  $y_n > 0$  for all  $n \in \mathbb{N}$  then  $x_n y_n$  converges.

## Question 2 - Convergence and Subsequences

(Wade 2.1.8/46) Suppose that  $\{x_n\}$  is a sequence in  $\mathbb{R}$ . Prove that  $x_n$  converges to  $a$  if and only if every subsequence of  $x_n$  also converges to  $a$ .

## Question 3 - Arithmetic of Convergence

Prove the following arithmetic properties of limits:

- 1 If  $x_n \rightarrow a$  and  $y_n \rightarrow b$ , then  $x_n + y_n \rightarrow a + b$  as  $n \rightarrow \infty$ .
- 2 If  $x_n \rightarrow a$ , then  $cx_n \rightarrow ca$  for any  $c \in \mathbb{R}$  as  $n \rightarrow \infty$ .
- 3 If  $x_n \rightarrow a$  and  $y_n \rightarrow b$ , then  $x_n y_n \rightarrow ab$  as  $n \rightarrow \infty$ .
- 4 If  $x_n \rightarrow a$  and  $y_n \rightarrow b$  and  $b \neq 0$  then  $\frac{x_n}{y_n} \rightarrow \frac{a}{b}$  as  $n \rightarrow \infty$ .