Tutorial 4

MATH3020: Real Analysis

3 - Density

4 - Mathematical Induction

Benjamin Fedoruk

Ontario Tech University

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Question 1 - Even and Odd Sequences

Suppose (x_n) is an infinite sequence of natural numbers, i.e., $(x_n) \in \mathbb{N}^{\mathbb{N}}$. Prove that (x_n) contains either an infinite subsequence of even numbers or an infinite subsequence of odd numbers.

Question 2 - Induction for Sum of Cubes Formula

Prove by induction that for all $n \in \mathbb{N}$, the sum of cubes of the first n natural numbers is given by:

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2} \right)^2$$

Question 3 - Lemma 1.25

Prove: If $n, k \in \mathbb{N}$ and $1 \leq k \leq n$ then,

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

Question 4 - Simplifying a Sum

Prove the following for all $n \in \mathbb{N}$:

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$$