

Tutorial 3

MATH3020: Real Analysis 3 - Infima and Suprema

Benjamin Fedoruk

Ontario Tech University

September 24, 2024

Question 1 - Finding Infima and Suprema

(Wade 1.3.1) Find the infimum and supremum of each of the following sets.

① $E = \{x \in \mathbb{R} : x^2 + 2x = 3\}$

② $E = \{x \in \mathbb{R} : x^2 - 2x + 3 > x^2 \text{ and } x > 0\}$

③ $E = \{x \in \mathbb{R} : x = \frac{1}{n} + (-1)^n \text{ for } n \in \mathbb{N}\}$

Question 2 - Infimum/Supremum True/False

(Wade 1.3.0) True or false:

- 1 If A and B are nonempty, bounded subsets of \mathbb{R} then $\sup(A \cap B) \leq \sup(A)$.
- 2 Let $\varepsilon \in \mathbb{R}_{>0}$. If $\mathbb{R} \supseteq A \neq \emptyset$ is bounded and $B := \{\varepsilon x : x \in A\}$ then $\sup B = \varepsilon \sup A$.
- 3 Let $\mathbb{R} \supseteq A, B \neq \emptyset$ be bounded. Then $\sup(A + B) = \sup(A) + \sup(B)$ and $\inf(A + B) = \inf(A) + \inf(B)$

Question 3 - Suprema/Infima of Finite Intersections

Let $\mathbb{R} \supseteq A_1, A_2, \dots, A_n \neq \emptyset$ be bounded sets. Then,

$$\sup \left(\bigcap_{i=1}^n A_i \right) \leq \min (\{\sup(A_i)\}_{i=1}^n)$$