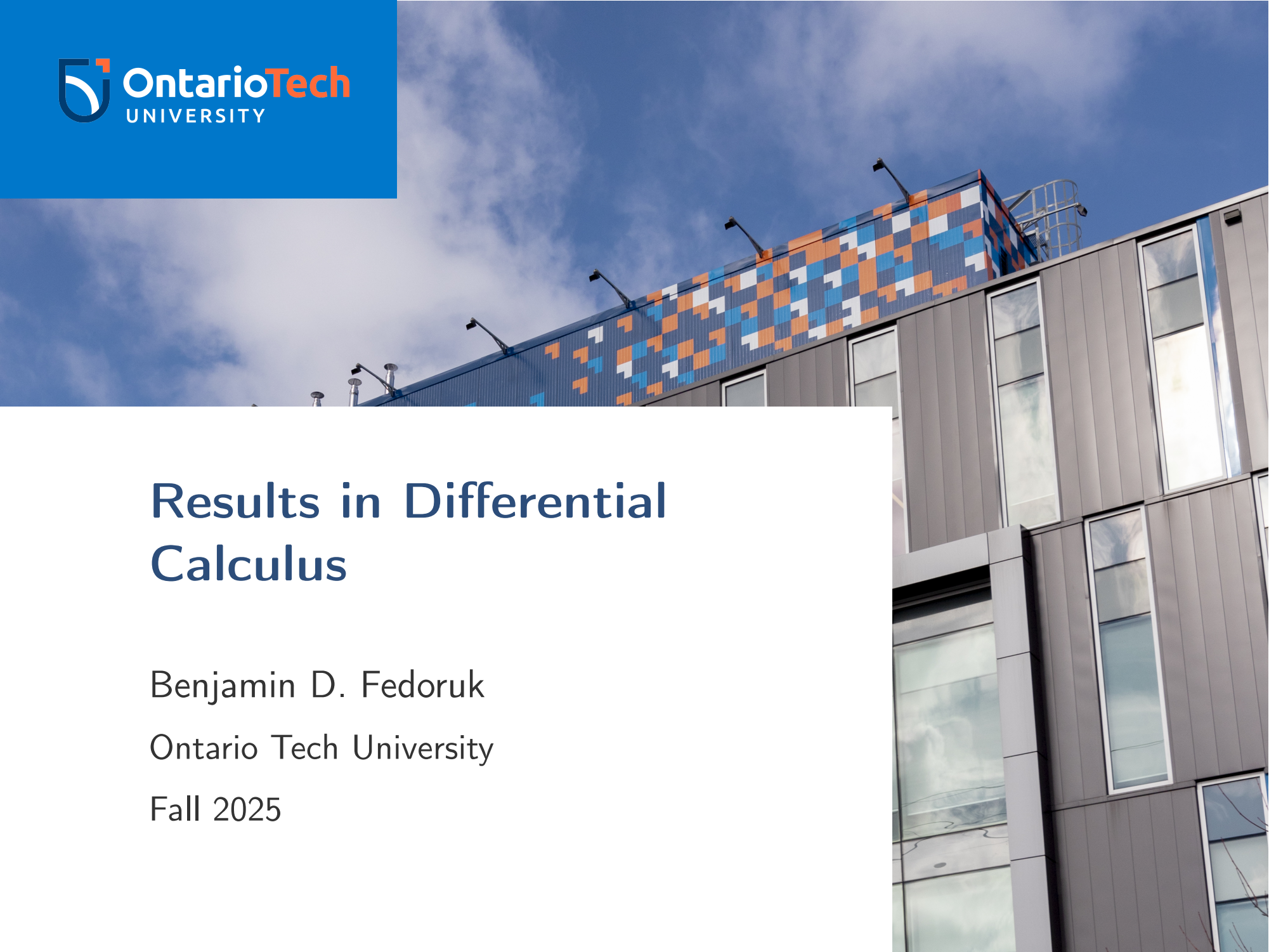

Results in Differential Calculus

Benjamin D. Fedoruk

Ontario Tech University

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Question 1 - Derivative of the Sinusoid

a) (As a group:) Prove that $\lim_{h \to 0} \frac{\sin h}{h} = 1$ without using l'Hoȋtal's Rule.

b) Prove that $\frac{\mathrm{d}}{\mathrm{d}x} \sin x = \cos x$.

Question 2 - Bizarre Proof of Power Rule

(Wade 4.2.3) Assume that $(e^x)' = e^x$ for $x \in \mathbb{R}$ and $(\log(x))' = \frac{1}{x}$ for $x > 0$. Use $x^\alpha := e^{\alpha \log x}$ to prove that $(x^\alpha)' = \alpha x^{\alpha-1}$ for all $x > 0$ and all $\alpha \in \mathbb{R}$.

Question 3 - Application of MVT

Use the Mean Value Theorem to prove that for $0 < a < b$,

$$\frac{b-a}{b} < \ln\left(\frac{b}{a}\right) < \frac{b-a}{a}$$