A SEQUENCE CONCEPTUALLY IS AN INFINITELY-LONG "LIST" OF NUMBERS

DEFINITIONALLY WE SAY (x_n) IS A <u>SEQUENCE</u> WHICH IS SOME FUNCTION FROM $\mathbb{N}$ TO $\mathbb{R}$. SO A SEQUENCE IS A FUNCTION WITH DOMAIN $\mathbb{N}$.

WE WRITE $(x_n) \in \mathbb{R}^{\mathbb{N}}$.

NOTE: THIS IS NOT A SET! A SET IS UNORDERED AND HAS NO DUPLICATES, A SEQUENCE IS ORDERED AND MAY HAVE REPITITION.

WE SAY A SEQUENCE IS <u>BOUNDED</u> IF $\forall n \in \mathbb{N}$, $|x_n| \leq M$ FOR SOME $M \in \mathbb{R}$.

WE SAY A SEQUENCE (x_n) IS <u>CONVERGENT</u> IF $\forall \varepsilon > 0$ $\exists N \in \mathbb{N}$ S.T. $\forall n > N$, $|x_n - a| < \varepsilon$.

I STRONGLY RECOMMEND YOU MEMORIZE THIS. JUST REPEAT IT OVER AND OVER AGAIN.

IF A SEQUENCE DOES NOT CONVERGE, WE SAY IT IS DIVERGENT.

WE WRITE $x_n \to a$ TO MEAN THAT THE SEQUENCE (x_n) CONVERGES TO $a \in \mathbb{R}$.

CONCEPTUALLY, $x_n \to a$ MEANS THE FOLLOWING:

IF YOU GIVE ME ANY $\varepsilon > 0$ (CAN'T BE 0), I CAN PICK SOME INDEX/INPUT N SUCH THAT AFTER THIS INDEX, ALL POINTS WILL WITHIN ε DISTANCE FROM a.

SOME NICE PROPERTIES OF CONVERGENCE:

- IF $x_n \to a$ AND $y_n \to b$ THEN $x_n + y_n \to a+b$
- IF $x_n \to a$ AND $y_n \to b$ THEN $x_n - y_n \to a-b$
- IF $x_n \to a$ AND $c \in \mathbb{R}$ THEN $cx_n \to ca$
- IF $x_n \to a$ AND $y_n \to b$ THEN $x_n y_n \to ab$
- IF $x_n \to a$, $y_n \to b$ AND $b \neq 0$ THEN $\frac{x_n}{y_n} \to \frac{a}{b}$.