

A SEQUENCE CONCEPTUALLY IS AN INFINITELY-LONG "LIST" OF NUMBERS
 PERMITIONALLY WE SAY (x_n) IS A SEQUENCE WHICH IS SOME FUNCTION FROM \mathbb{N} TO \mathbb{R} . SO A SEQUENCE IS A FUNCTION WITH DOMAIN ~~\mathbb{R}~~ \mathbb{N} .
 WE WRITE $(x_n) \in \mathbb{R}^{\mathbb{N}}$.

NOTE: THIS IS NOT A SET! A SET IS UNORDERED AND HAS NO DUPLICATES, A SEQUENCE IS ORDERED AND MAY HAVE REPETITION.

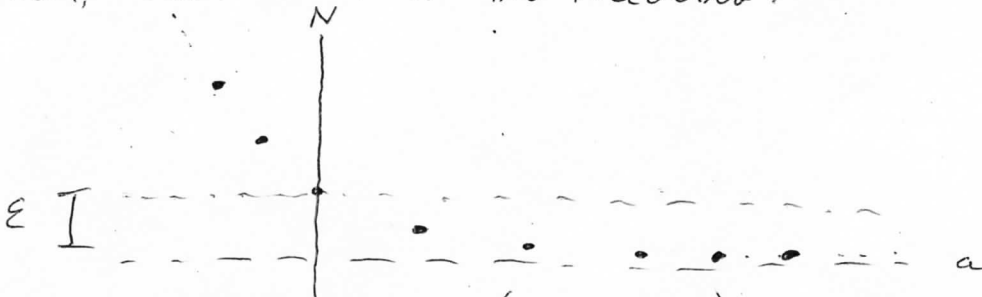
WE SAY A SEQUENCE IS BOUNDED IF $\forall n \in \mathbb{N}, |x_n| \leq M$ FOR SOME $M \in \mathbb{R}$.
 WE SAY A SEQUENCE (x_n) IS CONVERGENT IF $\forall \epsilon > 0 \exists N \in \mathbb{N}$ S.T. $\forall n > N, |x_n - a| < \epsilon$.

I STRONGLY RECOMMEND YOU MEMORIZE THIS. JUST REPEAT IT OVER AND OVER AGAIN.

IF A SEQUENCE DOES NOT CONVERGE, WE SAY IT IS DIVERGENT.

WE WRITE $x_n \rightarrow a$ TO MEAN THAT THE SEQUENCE (x_n) CONVERGES TO $a \in \mathbb{R}$.

CONCEPTUALLY, $x_n \rightarrow a$ MEANS THE FOLLOWING:



IF YOU GIVE ME ANY $\epsilon > 0$ (CAN BE 0), I CAN PICK SOME INDEX / INPUT N SUCH THAT AFTER THIS INDEX, ALL POINTS WILL WITHIN ϵ DISTANCE FROM a .

SOME NICE PROPERTIES OF CONVERGENCE:

- IF $x_n \rightarrow a$ AND $y_n \rightarrow b$ THEN $x_n + y_n \rightarrow a + b$
- IF $x_n \rightarrow a$ AND $y_n \rightarrow b$ THEN $x_n - y_n \rightarrow a - b$
- IF $x_n \rightarrow a$ AND $c \in \mathbb{R}$ THEN $cx_n \rightarrow ca$
- IF $x_n \rightarrow a$ AND $y_n \rightarrow b$ THEN $x_n y_n \rightarrow ab$
- IF $x_n \rightarrow a, y_n \rightarrow b$ AND $b \neq 0$ THEN $\frac{x_n}{y_n} \rightarrow \frac{a}{b}$.