WE'VE DONE INDUCTION BEFORE IN DISCRETE MATH.

<u>STRONG INDUCTION</u> — REQUIRES $P(1), P(2), \ldots, P(n)$ TO SHOW $P(n+1)$

<u>WEAK INDUCTION</u> — ONLY REQUIRES $P(n)$ TO SHOW $P(n+1)$.

THE <u>BINOMIAL THEOREM</u> SHOWS HOW WE CAN EXPAND BINOMIALS:

$$(x+y)^n = \sum_{i=1}^{n} \binom{n}{i} x^{n-i} y^i$$

WHERE $\binom{n}{i} = \frac{n!}{i!(n-i)!}$ IS THE <u>COMBINATORIAL</u> OPERATOR, READ AS "n CHOOSE i".

NOTE THE FOLLOWING USEFUL PROPERTY OF COMBINATORIAL OPERATORS:

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

<u>Pf.</u> LET $n, k \in \mathbb{N}$.

$$\binom{n+1}{k} = \frac{(n+1)!}{k!(n+1-k)!}$$

AND

$$\binom{n}{k-1} + \binom{n}{k} = \frac{n!}{(k-1)!(n-k+1)!} + \frac{n!}{k!(n-k)!}$$

$$= \frac{n!k}{k!(n-k+1)!} + \frac{(n-k+1)n!}{k!(n-k)!}$$

$$= \frac{n!k + n!(n-k+1)}{k!(n-k+1)!}$$

$$= \frac{n!(n+1)}{k!(n-k+1)!} = \frac{(n+1)!}{k!(n-k+1)!} = \binom{n+1}{k} \quad \square$$