LAST WEEK, WE TALKED ABOUT INVERSES OF 2×2 MATRICES. LET'S GENERALIZE! IF WE TAKE OUR $n\times n$ MATRIX, THEN AUGMENT WITH I, THE $n\times n$ IDENTITY, THEN WE CAN USE GAUSS-JORDAN ELIMINATION. ONCE THE LHS MATRIX IS I, THE RHS WILL BE THE INVERSE.

eg/

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 : R_2 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 5 & -2 & -2 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \\ 0 & 5 & -2 & -2 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 : R_2/-2} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 5 & -2 & -2 & 1 & 0 \end{array}\right] \xrightarrow{R_3 : R_3 - 5R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & -2 & 1 & \frac{5}{2} \end{array}\right]$$

$$\xrightarrow{R_2 : R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 2 \\ 0 & 0 & \frac{1}{2} & -2 & 1 & \frac{5}{2} \end{array}\right] \xrightarrow{R_3 : R_3 \times 2} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 2 \\ 0 & 0 & 1 & -4 & 2 & 5 \end{array}\right]$$

$$\xrightarrow{R_1 : R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & -2 & 1 & 2 \\ 0 & 0 & 1 & -4 & 2 & 5 \end{array}\right] \xrightarrow{R_1 : R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & -3 \\ 0 & 1 & 0 & -2 & 1 & 2 \\ 0 & 0 & 1 & -4 & 2 & 5 \end{array}\right] = [I|A^{-1}]$$

EVERY ELEMENTARY ROW OPERATION WE USE IN GAUSSIAN ELIMINATION (OR GAUSS-JORDAN) CAN BE REPRESENTED AS A SINGLE MATRIX. APPLYING THE OPERATION IS THE SAME AS LEFT-MULTIPLYING BY THE <u>ELEMENTARY MATRIX</u>

egs/

$R_1 : R_1/2$
$$E = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$
$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$R_3 : R_3 - 2R_1$
$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

SOME SPECIAL MATRICES:

<u>DIAGONAL MATRICES</u> HAVE NUMBERS ALONG MAIN DIAGONAL, ZEROS EVERYWHERE ELSE

<u>UPPER (LOWER) TRIANGULAR MATRICES</u> HAVE NUMBERS ON AND ABOVE (BELOW) THE MAIN DIAGONAL AND ZEROS EVERYWHERE ELSE.

<u>SYMMETRIC MATRICES</u> ARE ANY MATRICES WHERE $A^T = A$.

NOW WE WILL SHIFT TO TALK ABOUT A NEW CONCEPT CALLED THE <u>DETERMINANT</u>.
THE DETERMINANT IS RELATED TO AREA/VOLUME. I'LL TALK MORE ABOUT THIS LATER!

2×2 DETERMINANT:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

(LOOKS FAMILIAR!)

3×3 DETERMINANT:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + bfg + cdh - ceg - bdi - afh$$

TRICK FOR 3×3: ADD ALONG DIAGONALS, SUBTRACT ALONG ANTIDIAGONALS:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

WRITE TWICE

$$\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}$$

NOTE: THE DETERMINANT IS A NUMBER!

eg/ $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1\cdot 4 - 2\cdot 3 = 4-6 = -2$

eg/ $\begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 1\cdot 6 - 2\cdot 3 = 6-6 = 0$ (BECAUSE $\det(A) = 0 \Rightarrow A$ IS NOT INVERTIBLE)

eg/ $\begin{vmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \\ 2 & -1 & -4 \end{vmatrix} = -16 + 0 - 6 - 16 + 5 - 0 = -33$

WE CANNOT GENERALLY FIND THE DETERMINANT EASILY FOR $n \times n$ $n \geq 4$, NO TRICKS!
BUT WE CAN USE <u>COFACTOR EXPANSION</u>.
LET'S SHOW WITH AN EXAMPLE:

eg/
$$\begin{vmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \\ 2 & -1 & -4 \end{vmatrix} = (-1)^{1+1}(1)\begin{vmatrix} 4 & 5 \\ -1 & -4 \end{vmatrix} + (-1)^{1+3}(2)\begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix}$$
$$= \begin{vmatrix} 4 & 5 \\ -1 & -4 \end{vmatrix} + 2\begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix}$$
$$= -21 - 22 = -33$$