

# Tutorial 6

CSCI2110/MATH2080: Discrete Mathematics

**2.5 - Cardinality**

**3.1 - Algorithms**

**3.2 - The Growth of Functions**

**3.3 - Complexity of Algorithms**

**4.1 - Divisibility and Modular Arithmetic**

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October 22, 2024

## Question 1 - Countable Sets

Show that the following sets are countable:

- 1  $2\mathbb{N}$  (i.e., the even positive integers)
- 2  $2\mathbb{N} + 1$  (i.e., the odd positive integers)
- 3  $\mathbb{Z}$
- 4  $\mathbb{Q}$
- 5 The set of all Turing machines,  $T$ , where a Turing machine is any finite sequence of 0 and 1.

## Question 2 - Shaker Sort

(Rosen 11-13/246) The shaker sort algorithm successively compares pairs of adjacent elements, exchanging them if they are out of order, and alternately passing through the list from the beginning to the end and then from the end to the beginning until no exchanges are needed.

- 1 Show the steps used by the shaker sort to sort the list 3, 5, 1, 4, 6, 2.
- 2 Express the shaker sort algorithm in pseudocode or a programming language of your choice.
- 3 Show that the shaker sort algorithm has  $O(n^2)$  complexity measured in terms of the number of comparisons it uses.

## Question 3 - Products and the Modulus

(Rosen 42/259) Show that if  $a, b, c, m \in \mathbb{Z}$  such that  $m \geq 2$  and  $c > 0$  and  $a \equiv b \pmod{m}$  then  $ac \equiv bc \pmod{mc}$