

Tutorial 11

CSCI2110/MATH2080: Discrete Mathematics

9.1 - Relations and Their Properties

9.3 - Representing Relations

10.1 - Graphs and Graph Models

10.2 - Graph Terminology and Special Types of Graphs

10.3 - Representing Graphs and Graph Isomorphism

10.4 - Connectivity

10.5 - Euler and Hamilton Paths

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Question 1 - Some Difficult Graph Proofs

Select and prove one of the following:

- 1 **The Odd Degree Parity Theorem.** In any graph, the number of vertices with odd degree must be even.
- 2 **The Triangle-Free Theorem.** If G is a triangle-free, simple graph with n vertices, then G has at most $\frac{n^2}{4}$ edges.

Question 2 - Computation and Conjecture

(Rosen 3-8/466) Determine whether each of the following relations are reflexive, symmetric, antisymmetric, and/or transitive.

- 1 $R = \{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$ on $\{1, 2, 3, 4\}$.
- 2 $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$ on $\{1, 2, 3, 4\}$.
- 3 $R = \{(x, y) : x + y = 0\}$ on \mathbb{R} .
- 4 $R = \{(x, y) : x - y \in \mathbb{Q}\}$ on \mathbb{R} .
- 5 $R = \{(x, y) : x \equiv y \pmod{7}\}$ on \mathbb{Z} .
- 6 $R = \{(a, b) : a \text{ and } b \text{ have a shared grandparent}\}$ on the set of all people.
- 7 $R = \emptyset$ on a nonempty set S .