

Problem Set 2

Please complete and submit **any ONE** of the following problems. The deadline for submission is May 3, 2025 by 11:59 PM. Upload your submission here with the filename “ASP2-[Surname]”. Either typeset your solution using LaTeX or submit a clear PDF scan of your handwritten work. Clearly indicate which problem you are attempting to solve.

1. Show that the set of all points that are at most distance r from a convex set C is also a convex set.
2. Let $S = \{x_1, x_2, \dots, x_n\}$ be a set of points in $\mathbb{R}^d$. Prove that any point in the convex hull of S can be expressed as a convex combination of at most $d + 1$ points from S .
3. Prove or disprove: If C is a convex set in $\mathbb{R}^n$ and p is a point not in C then there exists a unique point in C that is closest to p .
4. Let P be a convex polytope in $\mathbb{R}^n$ defined as the convex hull of the points $\{v_1, \dots, v_m\}$. Prove that a point $p \in P$ is an extreme point if and only if $p \in \{v_1, \dots, v_m\}$.
5. Prove that a point p in a convex polytope P is an extreme point if and only if there exists a hyperplane H such that $H \cap P = \{p\}$.