Linear Transformations

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Tutorial 2
MATH 2055U (Advanced Linear Algebra)
Ontario Tech University

Winter 2026

Question 1 - Toy Problem of Linearity

(Axler 3.A.2) Suppose $b, c \in \mathbb{R}$. Define $T : \mathcal{P}(\mathbb{R}) \to \mathbb{R}^2$ by,

$$Tp = \left(3p(4) + 5p'(6) + bp(1)p(2), \int_{-1}^{2} x^3 p(x)dx + c\sin(p(0))\right)$$

Show that T is linear if and only if $b = c = 0$.

Question 2 - Existence of a Non-Commutator

(Axler 3.A.14) Suppose V is a finite-dimensional vector space with $\dim\left(()\, V\right) \geq 2$. Prove that there exist $S, T \in \mathcal{L}(V, W)$ such that $ST \neq TS$.

Question 3 - One-Dimensional Transformations

(Axler 3.A.7) Show that every linear map from a 1-dimensional vector space to itself is multiplication by some scalar. More precisely, prove that if $\dim\left(\left(\right)\right) V) = 1$ and $T \in \mathcal{L}(V, V)$, then there exists $\lambda \in \mathbb{K}$ such that $Tv = \lambda v$ for all $v \in V$.

Question 4 - Surjection and Dimension

(Axler 3.B.18) Suppose V and W are both finite-dimensional vector spaces. Prove that if there exists a surjective linear map $A : V \twoheadrightarrow W$ if and only if $\dim(V) \geq \dim(W)$,