

# Practice Problems

## MATH2055: Advanced Linear Algebra Tutorial 1 **Vector Spaces, Bases, and Transformations**

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## Question 1 - Deriving Sine and Cosine Sum Identities

(Treil 5.3/23) Using the rotation matrices for rotation by angles  $\alpha$  and  $\beta$  (i.e.,  $R_\alpha$  and  $R_\beta$ ) and matrix multiplication, derive the two identities for the sine and cosine of sums.

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

## Question 2 - Vector Spaces with One Basis

(Axler 2.B.1)

Find all vector spaces that have exactly one basis. (Hint: There are 2!)

## Question 3 - Equal Dimensions

(Axler 2.C.1)

Prove or give a counterexample: If  $V$  is a finite-dimensional vector space, and  $U$  is a subspace of  $V$  such that  $\dim(U) = \dim(V)$ , then  $U = V$ .