

Practice Problems

MATH2055: Advanced Linear Algebra Tutorial 9
Singular Values, SVD and PCA

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Question 1 - PCA Example

(As a group) Consider the following data about dog breeds. Perform PCA on this dataset and interpret the results.

Breed	Weight (kg)	Height (cm)	Life Span (yrs)
Golden Retriever	29.5	76	11
Great Dane	71.5	112	8.5
Chihuahua	2.25	52	13.5
Border Collie	17	52	13.5
Corgi	12	27.5	13.5

Question 2 - Singular Values True/False

For each of the following, determine if the statement is true or false. If true, prove it. If false, provide a counterexample.

- 1 Suppose $T \in \mathcal{L}(V)$. T is invertible if and only if all its singular values are nonzero.
- 2 Suppose $T \in \mathcal{L}(V)$. T and its transpose have the same singular values.
- 3 If $T \in \mathcal{L}(V)$ then the singular values of T^2 equal the squares of the singular values of T .

Question 3 - Computing the SVD

(Treil 6.3.5) Compute the singular value decomposition $A = U\Sigma V^T$ for the following matrices:

$$\textcircled{1} \quad A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$$

$$\textcircled{2} \quad A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$$

$$\textcircled{3} \quad A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$