Practice Problems

MATH2055: Advanced Linear Algebra Tutorial 6

The Determinant

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Question 1 - Multilinearity

If A is an $n \times n$ matrix and λ is a scalar prove that $\det((\lambda A)) = \lambda^n \det(A)$ using the formal definition of the determinant.

Question 2 - Cauchy-Binet Formula

(As a class) Prove the **Cauchy-Binet formula** using the permutation definition of determinants.
The Cauchy-Binet formula states:

$$\det((AB)) = \sum_S \det(A_S)\det(B_S)$$

where the sum is over all subsets $S \subset \{1, 2, ..., n\}$ with $|S| = m$, A_S (B_S) is the $m \times m$ submatrix of A (B) formed by selecting the columns (rows) indexed by S.

Question 2 - The Permanent

Consider the **permanent** of a matrix, defined similarly to the determinent but without the sign:

$$\text{perm}(A) := \sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i,\sigma(i)}$$

1. Express the permanent of a general 3×3 matrix explicitly.
2. Prove that for any matrix A, $|\det((A))| \leq \text{perm}(\text{abs}(A))$ where $\text{abs}(A)$ is the matrix of absolute values.