Practice Problems

MATH2055: Advanced Linear Algebra Tutorial 2

Dimension

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Question 1 - Vector Spaces with One Basis

(Axler 2.B.1)
Find all vector spaces that have exactly one basis. (Hint: There are 2!)

Question 2 - Equal Dimensions

(Axler 2.C.1)
Prove or give a counterexample: If V is a finite-dimensional vector space, and U is a subspace of V such that $\dim(U) = \dim(V)$, then $U = V$.

Question 3 - Infinite Dimension

In this exercise, we'll consider infinite-dimensional vector spaces. Below are two examples. Think about these two examples. How are they similar? How do they differ? Make a proposition about the subspaces of one of these examples and try to prove it.

$$\text{Vector Space 1:} \quad (\mathbb{R}^{\mathbb{N}}, +, \times)$$

$$\text{Vector Space 2:} \quad ((0,1], \times, \star)$$

(where $\star$ is the "scalar multiplication" defined by $\alpha \star v = v^{|\alpha|}$)