Practice Problems

MATH2055: Advanced Linear Algebra Tutorial 2

Invertibility and Isomorphisms

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January 23, 2025

Question 1 - Finding Isomorphisms

(Anton 8.3.9 and 8.3.10)

1. Show that $\mathbb{P}_2 \simeq \mathbb{R}^3$ by finding an isomorphism.
2. Let S_3 be the vector space of all 3×3 symmetric matrices. Show that $S_3 \simeq \mathbb{R}^6$ by finding an isomorphism.
3. Consider the vector space $V := \text{span}\{1, \sin t, \cos t\}$. Show that $V \simeq \mathbb{R}^3$ by finding an isomorphism.

Question 2 - Isomorphism is Transitive

(Anton 8.3.23) Prove that if U V, and W are vector spaces such that $U \simeq V$ and $V \simeq W$ then $U \simeq W$.

Question 2 - Injection and Dimension

(Axler 3.B.17) Suppose V and W are both finite-dimensional vector spaces. Prove that there exists an injective linear map $A : V \hookrightarrow W$ if and only if $\dim(V) \leq \dim(W)$.

Question 3 - Surjection and Dimension

(Axler 3.B.18) Suppose V and W are both finite-dimensional vector spaces. Prove that there exists a surjective linear map $A : V \twoheadrightarrow W$ if and only if $\dim(V) \geq \dim(W)$.