

Practice Problems

MATH2055: Advanced Linear Algebra Tutorial 1 **Vector Spaces, Bases, and Transformations**

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January 23, 2025

Question 1 - Deriving Sine and Cosine Sum Identities

(Treil 5.3/23) Using the rotation matrices for rotation by angles α and β (i.e., R_α and R_β) and matrix multiplication, derive the two identities for the sine and cosine of sums.

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

Question 2 - Extending a Basis

(Chal. Pr. 1.1) Let V be a finite-dimensional vector space and U be a subspace of V . Prove that any basis of U can be extended to a basis of V .