

Practice Problems

MATH2055: Advanced Linear Algebra Tutorial 4

Linear Transformations

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Question 1 - Linear Transformation on 1-Dimensional Vector Space

Let \mathbb{F} be an arbitrary field. Let V be a vector space over \mathbb{F} such that $\dim(V) = 1$. Let $T : V \mapsto V$ be a linear transformation. Show that $T : v \mapsto \lambda v$ for some scalar $\lambda \in \mathbb{F}$.

Question 2 - Linear Transformations as Matrices

(Treil 1.3.3c and 1.3.4a,b)

Find the matrix representation of the following linear transformations:

- 1 $P : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ such that P projects every vector onto the x, y -plane.
- 2 $R : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ such that R reflects every vector over the x, y -plane.
- 3 $D : \mathbb{P}_n \rightarrow \mathbb{P}^n$ such that $D : p \mapsto p'$.

Question 3 - Injection and Dimension

(Axler 3.B.17)

Suppose V and W are both finite-dimensional. Prove that there exists an injective linear map from V to W if and only if $\dim(V) \leq \dim(W)$.

Question 4 - Surjection and Dimension

(Axler 3.B.18)

Suppose V and W are both finite-dimensional. Prove that there exists a surjective linear map from V to W if and only if $\dim(V) \geq \dim(W)$.