Practice Problems

MATH2055: Advanced Linear Algebra Tutorial 3
Inner Product Spaces

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(Treil 5.1.5)

For each of the following, determine if the defined operation is an inner product on the given vector space:

- $\langle x, y \rangle = x_1 y_1 + x_2 y_2$ on $\mathbb{C}^2$.
- $\langle x, y \rangle = x_1 y_1 - x_2 y_2$ on $\mathbb{R}^2$.
- $\langle A, B \rangle = \operatorname{tr}(A + B)$ on $\mathbb{M}_{n \times n}$.

(Axler 6.A.4a)

Suppose that V is a real inner product space. Show that $\langle u + v, u - v \rangle = ||u||^2 - ||v||^2$ for every $u, v \in V$.

Bonus: Can you generalize this proof to the case when V is any inner product space (i.e., not necessarily real)?

(Treil 5.1.4)

Prove that for vectors in an inner product space:

$$||x \pm y||^2 = ||x||^2 + ||y||^2 \pm \text{Re}\langle x, y \rangle$$