Practice Problems

MATH2055: Advanced Linear Algebra Tutorial 2

Basis and Dimension

Benjamin Fedoruk

Ontario Tech University

January 23, 2025

Question 1 - Basis of Antisymmetric Matrices

(Treil 1.2.4c)
Recall that an antisymmetric matrix is any matrix A which satisfies $A^\top = -A$. Find a basis for:

- The set of 2×2 antisymmetric matrices.
- The set of 3×3 antisymmetric matrices.
- The set of $n \times n$ antisymmetric matrices (for $n \in \mathbb{N}$).

Question 2 - On Linear Independence

(Treil 1.2.6)
Is it possible that vectors v_1, v_2, v_3 are linearly dependent, but the vectors $w_1 = v_1 + v_2, w_2 = v_2 + v_3, w_3 = v_3 + v_1$ are linearly independent?

Question 3 - Equal Dimensions

(Axler 2.C.1)
Prove or give a counterexample: If V is a finite-dimensional vector space, and U is a subspace of V such that $\dim(U) = \dim(V)$, then $U = V$.

Question 4 - Vector Spaces with One Basis

(Axler 2.B.1)
Find all vector spaces that have exactly one basis. (Hint: There are 2!)