Practice Problems

MATH2055: Advanced Linear Algebra Tutorial 11

Singular Value Decomposition

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April 4, 2024

Question 1 - Singular Values True/False

For each of the following, determine if the statement is true or false. If true, prove it. If false, provide a counterexample.

1. Suppose $T \in \mathcal{L}(V)$. T is invertible if and only if all its singular values are nonzero.
2. Suppose $T \in \mathcal{L}(V)$. T and its transpose have the same singular values.
3. If $T \in \mathcal{L}(V)$ then the singular values of T^2 equal the squares of the singular values of T.

Question 2 - Computing the SVD

(Treil 6.3.5) Compute the singular value decomposition $A = U\Sigma V^\top$ for the following matrices:

1. $A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$
2. $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$
3. $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$