Practice Problems

MATH2055: Advanced Linear Algebra Tutorial 1

Proof and Vector Space Basics

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Question 1 - Extended Real Numbers

(Axler 1.B.6)
Let $\overline{\mathbb{R}} := \mathbb{R} \cup \{-\infty, \infty\}$ be the **extended real numbers**, the set of all real numbers along with $\pm\infty$. Here, addition works as you would expect. Note that $-\infty + \infty = 0$.
Does $\overline{\mathbb{R}}$ form a real vector space?

Question 2 - An Unusual Vector Space

We know that $\mathbb{R}$ with the usual addition and scalar multiplication form a real vector space. But what if we redefine the operations?
Consider $\mathbb{R}$ such that for any $u, v \in \mathbb{R}$, $u + v := \max(u, v)$ and scalar multiplication is defined in the typical way.
Does $\mathbb{R}$ under this new operator form a vector space?

Question 3 - Intersection of Subspaces

(Axler 1.C.10)
Suppose U_1 and U_2 are subspaces of V. Prove that the intersection $U_1 \cap U_2$ is also a subspace of V.