TODAY, I WANT TO INTRODUCE A FEW EXTENSIONS OF THE MATRIX PRODUCT.
IN REAL NUMBERS, WE DEFINE THE EXPONENTIAL USING THE TAYLOR SERIES.

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = \exp(x)$$

WE FOLLOW A SIMILAR DEFINITION TO DEFINE e^A FOR MATRIX A.

$$e^A := \sum_{n=0}^{\infty} \frac{A^n}{n!} = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

NOW LET'S INTRODUCE A COUPLE OTHER TYPES OF MATRIX PRODUCT:

1. <u>KRONECKER PRODUCT</u>

WE DEFINE THE KRONECKER PRODUCT $A \otimes B$ OF TWO MATRICES TO BE

$$A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix}$$

SO IF A IS $m \times n$ AND B IS $k \times l$ THEN $A \otimes B$ IS $km \times ln$.

eg/ $A = \begin{bmatrix} 2 & 1 \\ 5 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$

$$\Rightarrow A \otimes B = \begin{bmatrix} 2 & 1 & -4 & -2 \\ 5 & 4 & -10 & -8 \\ 6 & 3 & -8 & -4 \\ 15 & 12 & -20 & -16 \end{bmatrix}$$

2. <u>SCHUR PRODUCT</u>

WE DEFINE THE SCHUR PRODUCT (OR BAD STUDENTS' PRODUCT) $A \circ B$ TO BE:

$$(A \circ B)_{ij} = A_{ij} B_{ij}$$

eg/

$$A = \begin{bmatrix} 2 & 1 & 3 & 2 \\ 1 & 2 & 3 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 4 & 3 & 1 \\ 1 & 3 & 1 & 4 \\ 2 & 1 & 3 & 2 \end{bmatrix}$$

$$A \circ B = \begin{bmatrix} 4 & 4 & 9 & 2 \\ 1 & 6 & 3 & 8 \\ 6 & 1 & 6 & 6 \end{bmatrix}$$

WHAT ARE THE IDENTITIES FOR THE KRONECKER AND SCHUR PRODUCTS?
THAT IS, FIND $I_\circ$ SO THAT $I_\circ \circ A = A \circ I_\circ = A$
AND FIND $I_\otimes$ SO THAT $I_\otimes \otimes A = A \otimes I_\otimes = A$