THIS WEEK, WE DISCUSS ISOMORPHISMS OF VECTOR SPACES.

FIRST, IF WE HAVE A LINEAR TRANSFORMATION THAT IS LINEAR AND INVERTIBLE, THEN WE SAY IT IS AN ISOMORPHISM.

WHAT DOES IT MEAN FOR $T: U \to U$ TO BE LINEAR?

(a) T IS ADDITIVE, ie $T(u_1 + u_2) = T(u_1) + T(u_2)$ $\forall u_1, u_2 \in U$

(b) T IS HOMOGENEOUS, ie $T(ku_1) = kT(u_1)$ $\forall u_1 \in U, k \in \mathbb{F}$.

WHAT DOES IT MEAN FOR T TO BE INVERTIBLE?

WELL, T IS INVERTIBLE IFF IT IS BIJECTIVE.

WHAT DOES IT MEAN FOR T TO BE BIJECTIVE?

(a) T IS INJECTIVE, ie $T(u_1) = T(u_2) \Rightarrow u_1 = u_2$

(b) T IS SURJECTIVE, ie $\text{cod}(T) = \text{ran}(T)$.

NOTE: INJECTIVE IS ONE-TO-ONE AND SURJECTIVE IS ONTO.

ALSO NOTE THE PROPER DEFINITION OF AN INVERTIBLE T:

T IS INVERTIBLE IF THERE EXISTS SOME $S: V \to U$ SUCH THAT $T \circ S = id$, WHERE id IS THE IDENTITY MAP.

THIS CIRCLE REPRESENTS COMPOSITION, SO $(T \circ S)(x) = T(S(x))$. IT IS NOT COMMUTATIVE.

WE SAY TWO VECTOR SPACES U AND V ARE ISOMORPHIC IF WE CAN FIND AN ISOMORPHISM FROM U TO V (WHICH ALSO IMPLIES AN ISOM. FROM V TO U.) WE WRITE $U \simeq V$.

ISOMORPHISMS ARE HANDY AS THEY ALLOW YOU TO "TRANSLATE" A PROBLEM IN A HARD VECTOR SPACE U TO AN EASIER VECTOR SPACE V.

TYPICAL WORKFLOW: