THIS TIME WE'LL PUT "LINEAR" IN LINEAR ALGEBRA

WE SAY A MAP $T: V \to W$ <u>LINEAR</u> IF IT IS HOMOGENEOUS AND ADDITIVE.

THE MAP IS <u>ADDITIVE</u> IF $T(u+v) = T(u) + T(v) \ \forall u, v \in V$.

THE MAP IS <u>HOMOGENEOUS</u> IF $T(ku) = kT(u) \ \forall u \in U, k \in \mathbb{F}$

AN OPERATOR IS A TRANSFORM $T: U \to U$

WE CAN WRITE A LINEAR TRANSFORM AS A MATRIX!

eg/ CONSIDER $T: \mathbb{R}^2 \to \mathbb{R}^2$ SUCH THAT T REFLECTS A VECTOR OVER $y = x$.

THIS IS LIKE SWAPPING x AND y COMPONENTS.

TO WRITE THE MATRIX, TAKE BASIS AND APPLY TRANSFORM TO EACH BASIS VECTOR $e_1, \ldots, e_n$.

$$\begin{array}{l} T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\1\end{bmatrix} \\ T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix} \end{array} \xrightarrow[\text{MATRIX}]{\text{WRITE AS}} \begin{array}{cc} \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \\ \uparrow \quad \uparrow \\ T(e_1) \ T(e_2) \end{array} = [T].$$

THE SET OF LINEAR TRANSFORMS FROM V TO W IS <u>$\mathcal{L}(V,W)$</u>.

FOR ANY $T \in \mathcal{L}(V,W)$, THE <u>KERNEL</u> OF T IS THE SET OF VECTORS WHICH GET MAPPED TO 0 UNDER T. IT IS DENOTED $\ker(T)$

INTERESTINGLY, $\ker(T)$ IS A SUBSPACE OF V.

FOR ANY $T \in \mathcal{L}(V,W)$, THE <u>IMAGE</u> OF T IS THE SET OF VECTORS THAT "CAN BE REACHED" BY T. IT IS DENOTED $\mathrm{im}(T)$.

THE <u>NULLITY</u> OF T IS $\mathrm{nullity}(T) := \dim(\ker(T))$

THE <u>RANK</u> OF T IS $\mathrm{rk}(T) := \dim(\mathrm{im}(T))$.

THE <u>RANK-NULLITY THEOREM</u> SAYS: $\dim(V) = \mathrm{rk}(T) + \mathrm{nullity}(T)$

A FUNCTION $f: A \to B$ HAS <u>DOMAIN</u> A AND <u>CODOMAIN</u> B. BUT f MIGHT NOT "REACH" ALL OF B.

eg/ $f(x) = x^2$. MAYBE $f: \mathbb{R} \to \mathbb{R}$, SO $\mathrm{cod}(f) = \mathbb{R}$, BUT $\mathrm{im}(f) = \mathbb{R}_{\geq 0}$.

IF A FUNCTION "REACHES" ITS WHOLE CODOMAIN, ie IF $\mathrm{cod}(f) = \mathrm{im}(f)$, WE CALL IT <u>SURJECTIVE</u> (OR ONTO).

IF A FUNCTION SATISFIES $f(x) = f(y) \Rightarrow x = y$ THEN IT IS <u>INJECTIVE</u> OR 1-1.

IF A FUNCTION IS INJECTIVE AND SURJECTIVE, THEN IT IS <u>BIJECTIVE</u>.