I'M GOING TO START WITH SOMETHING A LITTLE ABSTRACT.

SAY WE HAVE n "SYMBOLS": $x_1, x_2, \ldots, x_n$. WE SAY THAT $\alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n$ REPRESENTS ANY <u>LINEAR COMBINATION</u> OF THE SYMBOLS, WHERE $\alpha_i \in \mathbb{F}$.

THE SET OF ALL VECTORS WHICH CAN BE REACHED USING A LINEAR COMBO OF A SET OF VECTORS IS CALLED THE <u>SPAN</u> OF THOSE VECTORS.

ie $\text{span}\{v_1, v_2, \ldots, v_n\} = \{\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n : \alpha_i \in \mathbb{F} \ \forall i \in \{1, \ldots, n\}\}$

IN PAST COURSE, WE SAID A SET WAS LINEARLY INDEPENDENT IF v_i COULD NOT BE WRITTEN AS A LINEAR COMBO OF THE OTHER VECTORS.

IN THIS COURSE, WE SAY $\{v_1, v_2, \ldots, v_n\}$ IS LINEARLY INDEPENDENT IFF $\alpha_1 \bar{v}_1 + \alpha_2 \bar{v}_2 + \cdots + \alpha_n \bar{v}_n = \bar{0} \iff \alpha_1 = \alpha_2 = \cdots = \alpha_n = 0$. (SO, THIS MUST BE THE <u>ONLY</u> SET OF CONSTANTS WHICH SATISFY THE EQUATION!)

RECALL THAT A SET OF VECTORS IS A <u>BASIS</u> FOR A VECTOR SPACE V IFF IT IS LINEARLY INDEPENDENT <u>AND</u> IT SPANS THE WHOLE SPACE.

THE DIMENSION IS THE # OF VECTORS IN ANY BASIS.

IT TURNS OUT THAT IT DOESN'T MATTER WHICH BASIS YOU CHOOSE! ALWAYS THE SAME # OF VECTORS IN ANY BASIS.

A GOOD WAY TO THINK ABOUT DIMENSION:

- A SET OF VECTORS WILL ALWAYS START OUT LINEARLY INDEPENDENT (CONSIDER $\{v\}$, A SINGLETON), AND AS WE ADD VECTORS TO THE SET, IT WILL BECOME LINEARLY DEPENDENT AFTER A "MAGIC # OF VECTORS" HAVE BEEN ADDED.
- A SET OF VECTORS WILL START OUT AS A SPAN "SMALLER" THAN THE VECTOR SPACE ITSELF. OVER TIME, IF WE ADD MORE VECTORS, THE "SIZE" OF THE SPAN WILL PLATEAU AT A "MAGIC # OF VECTORS"

THIS "MAGIC # OF VECTORS" IS THE DIMENSION!

NOTE: IT IS POSSIBLE TO HAVE AN ∞-DIMENSIONAL VECTOR SPACE! CONSIDER $\mathbb{R}^{\mathbb{R}}$.

(WHERE $\mathbb{R}^{\mathbb{R}} = \{f : \mathbb{R} \to \mathbb{R}\}$