05 - Universal Trigonometric Substitution We shall now examine the universal trigonometric substitution, a powerful technique for evaluating integrals involving rational functions of

trigonometric expressions.

Definition 1 (Universal Trigonometric Substitution)

Let
$$t = \tan\left(\frac{x}{2}\right)$$
. Then,

$$\sin x = \frac{2t}{1+t^2}, \qquad \cos x = \frac{1-t^2}{1+t^2}, \qquad dx = \frac{2}{1+t^2} dt$$

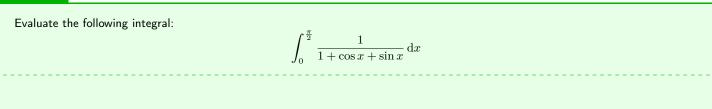
This substitution transforms any rational trigonometric integrand into a rational function of t, which can then be integrated using partial fraction decomposition.

Example 1

Evaluate the following integral:

$$\int_0^\pi \frac{1}{a + \cos x} \, \mathrm{d}x \qquad a > 1$$

Example 2



Example 3

Evaluate the following integral:	$\int_0^{\frac{\pi}{2}} \frac{1}{a\sin^2 x + b\cos^2 x} \mathrm{d}x$