

05 - Universal Trigonometric Substitution

We shall now examine the universal trigonometric substitution, a powerful technique for evaluating integrals involving rational functions of trigonometric expressions.

Definition 1 (Universal Trigonometric Substitution)

Let $t = \tan\left(\frac{x}{2}\right)$. Then,

$$\sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}, \quad dx = \frac{2}{1+t^2} dt$$

This substitution transforms any rational trigonometric integrand into a rational function of t , which can then be integrated using partial fraction decomposition.

Example 1

Evaluate the following integral:

$$\int_0^\pi \frac{1}{a + \cos x} dx \quad a > 1$$

Example 2

Evaluate the following integral:

$$\int_0^{\frac{\pi}{2}} \frac{1}{1 + \cos x + \sin x} dx$$

Example 3

Evaluate the following integral:

$$\int_0^{\frac{\pi}{2}} \frac{1}{a \sin^2 x + b \cos^2 x} dx$$
