

03 - Integrating Inverse Functions

We shall start this lecturette with a reminder of the strategy for integrating $\ln(x)$.

Example 1

Evaluate the following integral:

$$\int_1^e \ln(x) \, dx$$

Now we will extend this idea to be able to integrate any inverse function.

Theorem 1 (Laisant's Formula)

For any invertible function $f(x)$,

$$\int f^{-1}(x) \, dx = x f^{-1}(x) - (F \circ f^{-1})(x) + C$$

Proof.

□

For the following example, note that $\operatorname{erf}^{-1} x$ is the inverse error function such that if $x = \operatorname{erf} y$ then $y = \operatorname{erf}^{-1} x$.

Example 2

Evaluate the following integral:

$$\int_0^a \operatorname{erf}^{-1} x \, dx$$
