# 01 - Even-Odd Decomposition

Now, we are going to learn a very niche method of integration. That said, any method of integration which helps us to solve integrals is interesting and useful. Our new method is called "even-odd decomposition". Recall that an even function is a function such that $f(-x) = f(x)$ and an odd function is a function such that $f(-x) = -f(x)$. Recall the following properties of even and odd integrands:

If $f_o(x)$ is an odd function, then $\displaystyle\int_{-a}^{a} f_o(x)\,\mathrm{d}x = 0$.

If $f_e(x)$ is an even function, then $\displaystyle\int_{-a}^{a} f_e(x)\,\mathrm{d}x = 2\int_{0}^{a} f_e(x)\,\mathrm{d}x$.

You might not know this little fact, but it is always possible to decompose any function into the sum of an even function and an odd function. That is, we can find an even function $f_e(x)$ and an odd function $f_o(x)$ for any given function $f(x)$ so that $f(x) = f_e(x) + f_o(x)$. Below, I'll demonstrate how we find these two functions:

We will now evaluate an integral to showcase how this decomposition can help us in our quest to be able to tackle more integrals.

**Example 1**

Evaluate the following integral.

$$\int_{-1}^{1} \frac{\cos x}{e^{\frac{1}{x}} + 1}\,\mathrm{d}x$$