Tutorial 2

MATH3020: Real Analysis

2 - Metrics, Absolute Value and Topology of $\mathbb{R}$

Benjamin Fedoruk

Ontario Tech University

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Question 1 - Positive and Negative Parts

(Wade 1.2.3). The *positive part* of $a \in \mathbb{R}$ is defined by,

$$a^+ := \frac{|a| + a}{2}$$

and the *negative part* by

$$a^- := \frac{|a| - a}{2}$$

Prove that $a = a^+ - a^-$ and $|a| = a^+ + a^-$.

Question 2 - Proof Using Inequalities

True or false: $-3 \leq x \leq 2$ implies $|x^2 + x - 6| \leq 6|x - 2|$.

Question 3 - Algebraic and Geometric Means

The *arithmetic mean* of $a, b \in \mathbb{R}$ is $A(a,b) = \frac{a+b}{2}$ and the *geometric mean* of $a, b \in [0, \infty)$ is $G(a,b) = \sqrt{ab}$. If $0 \leq a \leq b$, prove that $a \leq G(a,b) \leq A(a,b) \leq b$. Prove that $G(a,b) = A(a,b)$ if and only if $a = b$.

Question 4 - Products of Sums of Squares

Prove that $(ab + cd)^2 \leq (a^2 + c^2)(b^2 + d^2)$ for all $a, b, c, d \in \mathbb{R}$.