Tutorial 5

MATH1850: Linear Algebra for Engineers

1.8 - Matrix Transformations
4.9 - Basic Matrix Transformations in $\mathbb{R}^2$ and $\mathbb{R}^3$
4.10 - Properties of matrix Transformations
5.1 - Eigenvalues and Eigenvectors

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Question 1 - Finding Standard Matrices for Transformations

For each of the following transformations, find the standard matrix T for the operator T:

1. $T(x_1, x_2) = (2x_1 - x_2, x_1 + x_2)$
2. $T(x_1, x_2) = (x_1, x_2)$
3. $T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, x_1 + 5x_2, x_3)$
4. $T(x_1, x_2, x_3) = (4x_1, 7x_2, -8x_1)$

Question 2 - Standard Matrix of Basic Matrix Transformations

Find the standard matrix for the following transformations:

1. $T : \mathbb{R}^2 \to \mathbb{R}^2$ where T reflects about the x-axis.
2. $T : \mathbb{R}^2 \to \mathbb{R}^2$ where T reflects about the y-axis.
3. $T : \mathbb{R}^2 \to \mathbb{R}^2$ where T reflects about the line $y = x$.
4. $T : \mathbb{R}^2 \to \mathbb{R}^2$ where T projects onto the x-axis.
5. $T : \mathbb{R}^2 \to \mathbb{R}^2$ where T projects onto the y-axis.
6. $T : \mathbb{R}^3 \to \mathbb{R}^3$ where T reflects about the xy-plane.
7. $T : \mathbb{R}^3 \to \mathbb{R}^3$ where T reflects about the xz-plane.
8. $T : \mathbb{R}^2 \to \mathbb{R}^2$ where T rotates about the origin by $30°$ counterclockwise.

Question 3 - Transformation Composition

Find the standard transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that results from first reflecting about the y-axis, then stretching by a factor of 3, then rotating about the origin by 30° counterclockwise, then projecting onto the x-axis.

Question 4 - Finding Eigenvalues and Eigenvectors

Find all eigenvalues and eigenvectors for the following matrices:

- $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$
- $B = \begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}$
- $C = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$
- $D = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

Question 5 - Zero Eigenvalues

Which of the following matrices have zero as an eigenvalue? You may use MATLAB/Octave, but do NOT use `eig`.

$$A = \begin{bmatrix} 2 & 0 & 1 & 3 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 3 & 2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 & 0 & 2 \\ 3 & 2 & 1 & 1 \\ 1 & 0 & 3 & 2 \\ 2 & 1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 4 & 5 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 0 & 4 & 2 \end{bmatrix}$$