Tutorial 4

MATH1850: Linear Algebra for Engineers

4.3 - Linear Independence
4.4 - Coordinates and Basis
4.5 - Dimension
4.7 - Rowspace, Columnspace and Nullspace
4.8 - Rank, Nullity and the Fundamental Matrix Spaces

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Question 1 - Linear Independence with Parameters

Determine all values of k for which the following matrices are independent in $\mathbb{M}_{22}$:

$$\left\{ \begin{bmatrix} 1 & 0 \\ 1 & k \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ k & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \right\}$$

Question 3 - Span of a Set of Vectors

Determine if each of the following sets of vectors spans the vector space given

1. $\{(-3, 0, 4), (5, -1, 2), (1, 1, 3)\}$ in $\mathbb{R}^3$
2. $\{(-2, 0, 1), (3, 2, 5), (6, -1, 1), (7, 0, -2)\}$ in $\mathbb{R}^3$
3. $\{2 - x + 4x^2, 3 + 6x + 2x^2, 2 + 10x - 4x^2\}$ in $\mathbb{P}_2$
4. $\{1 + 3x + 3x^2, x + 4x^2, 5 + 6x + 3x^2, 7 + 2x - x^2\}$ in $\mathbb{P}_2$

Question 4 - Basis or Not?

Which of the following sets form a basis for the given vector space? State the dimension of the span of the set.

1. $\{(3, 1, -4), (2, 5, 6), (1, 4, 8)\}$ in $\mathbb{R}^3$
2. $\{1 - 3x + 2x^2, 1 + x + 4x^2, 1 - 7x\}$ in $\mathbb{P}_2$
3. $\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$ in $\mathbb{M}_{22}$

Question 5 - Hermite Polynomials

The first four Hermite polynomials are $1, 2t, -2+4t^2, -12t+8t^3$. These polynomials have a wide variety of applications in physics and engineering.

1. Show that the first four Hermite polynomials form a basis for $\mathbb{P}_3$.
2. Let B be the basis in part (a). Find the coordinate vector of the polynomial $p(t) = -1 - 4t + 8t^2 + 8t^3$.

Question 6 - Bases for Rowspace, Columnspace and Nullspace of A

Find the bases for the rowspace, columnspace, and nullspace of each matrix. Find the rank and the nullity for each.

1. $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$
2. $B = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$