Tutorial 3

MATH1850: Linear Algebra for Engineers

3.3 - Orthogonality
3.4 - The Geometry of Linear Systems
3.5 - Cross Product
4.1 - Real Vector Spaces
4.2 - Subspaces

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Question 1 - Find an Orthonormal Vector

Find a vector which is orthonormal to $v = \begin{bmatrix} 4 \\ -3 \\ 2 \\ 6 \end{bmatrix}$.

Question 2 - Equation of a Plane and Cross Product

Find the equation of the plane in standard form which contains the vectors $u = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ and $w = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ and passes through the origin (i.e., the point $(0, 0, 0)$).

Question 3 - Vector Space Proofs or Contradictions I

Determine if each of the following sets equipped with the given operations forms a vector space.

1. The set of all pairs of numbers of the form $(a, 0)$ with standard addition and scalar multiplication.
2. The set of all n-tuples of real numbers that have the form $(x, x, ..., x)$ with standard addition and scalar multiplication.
3. The set of all vectors in $\mathbb{R}^2$ with addition $u + v = (u_1 + v_1 + 1, u_2 + v_2 + 1)$ and standard scalar multiplication.
4. The set of all 2×2 invertible matrices with the standard matrix addition and scalar multiplication.
5. The set of all real-valued functions f defined everywhere on the real line such that $f(1) = 0$ with $(f + g)(x) = f(x) + g(x)$ and $(kf)(x) = kf(x)$.

Question 4 - Vector Space Proofs or Contradictions II

Determine if each of the following sets equipped with the given operations forms a vector space.

6. The set of all polynomials of the form $a + bx$ with the operations $(a + bx) + (c + dx) = (a + c) + (b + d)x$ and $k(a + bx) = (ka) + (kb)x$.
7. The set of all points lying on the line $(x, y, z) = (1, 0, 0) + (1, 1, 1)t$ with standard addition and scalar multiplication.
8. The set $\{0\}$ containing only the zero vector where $0 + 0 = 0$ and $k \cdot 0 = 0$.
9. The set R^∞ of all infinite sequences of real numbers with operations $(x_1, x_2, ...) + (y_1, y_2, ...) = (x_1 + y_1, x_2 + y_2, ...)$ and $k(x_1, x_2, ...) = (kx_1, kx_2, ...)$.

Question 5 - The Subspace Test

Which of the following are subspaces of $\mathbb{M}_{2\times 2}$ (the vector space of 2×2 matrices):

1. The set of all diagonal 2×2 matrices.
2. The set of all singular 2×2 matrices.
3. The set of all 2×2 matrices with trace 0.
4. The set of all symmetric 2×2 matrices.
5. The set of all invertible 2×2 matrices.