Tutorial 2

MATH1850: Linear Algebra for Engineers

1.5 - Inverses by Row Reduction
1.6 - More on Linear Systems
1.7 - Diagonal, Triangular, and Symmetric Matrices
1.9 - Applications of Linear Systems
2.1 - Cofactor Expansion
2.2 - Determinants by Row Reduction
2.3 - Cramer's Rule
3.1 - Euclidean Vectors
3.2 - Norm, Dot Product, and Distance

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Question 1 - Inverses by Row Reduction

Find the inverse of the following matrix using row reduction:

$$A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$$

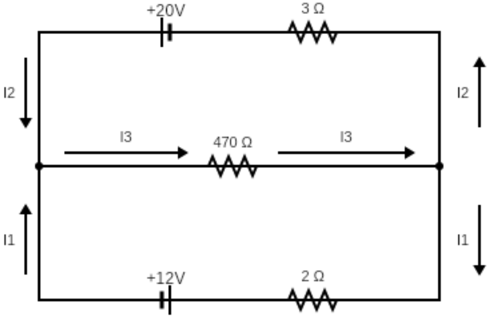
Check your result using the `inv` command in MATLAB/Octave.

Question 2 - Proof of Symmetry

Prove that if A is symmetric, then $A^{-1}A^{\top}$ is symmetric. (Try to write it in a formal proof format, as best you can.)

Question 3 - Resistor Networks

Determine the unknown currents in the accompanying circuit. Use MATLAB/Octave – this one is hard to do by hand!



Question 4 - 3×3 Trick for Determinants

Find the determinant of the following matrix using the 3×3 trick:

$$B = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}$$

Verify your result using the `det` method in MATLAB/Octave.

Question 5 - Cofactor Expansion

Find the determinant of the following matrix using cofactor expansion:

$$C = \begin{bmatrix} -1 & 0 & 0 & -2 \\ 1 & 0 & 5 & -5 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -5 & 0 \end{bmatrix}$$

Verify your result using the `det` command in MATLAB/Octave.

Question 6 - Determinant by Row Reduction

Using the same matrix as before, find the determinant using cofactor expansion.

$$C = \begin{bmatrix} -1 & 0 & 0 & -2 \\ 1 & 0 & 5 & -5 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -5 & 0 \end{bmatrix}$$

Question 7 - Cramer's Rule

Solve the following system using Cramer's Rule. Use the `det` command in MATLAB/Octave to save time.

$$\begin{aligned} x \quad -4y \quad +z \quad &= 6 \\ 4x \quad -y \quad +2z \quad &= -1 \\ 2x \quad +2y \quad -3z \quad &= -20 \end{aligned}$$