RECALL THAT THE STANDARD BASIS VECTORS $\hat{e}_1, \ldots, \hat{e}_n$ HAVE A 1 IN THE n^{TH} POSITION AND ZEROS EVERYWHERE ELSE.

WE'RE GOING TO TOUCH ON SOME DISCRETE MATH. LET'S TALK ABOUT FUNCTIONS, DOMAIN, IMAGE AND CODOMAIN.

CONSIDER $f(x) = x^2$. NOW $Dom(f) = \mathbb{R}$, $Cod(f) = \mathbb{R}$, BUT $Ran(f) = \mathbb{R}_{\geq 0}$

ANOTHER EXAMPLE: $g(x) = \sin(x)$. HERE $Dom(g) = Cod(g) = \mathbb{R}$ BUT $Ran(g) = (-1, 1)$.

TRANSFORMATIONS FROM $\mathbb{R}^n$ TO $\mathbb{R}^m$ ARE CALLED MATRIX TRANSFORMS.

WE CAN REPRESENT THEM IN A MATRIX BY SHOWING WHERE THE TRANSFORMATIONS TAKE $\hat{e}_1, \ldots, \hat{e}_n$

SOME "EASY" TRANSFORM EXAMPLES:

<u>THE ZERO TRANSFORM:</u>

$$T_0(x) = 0x = 0 \quad \forall x \in \mathbb{R}^n$$

<u>THE IDENTITY TRANSFORM:</u>

$$T_1(x) = Ix = x \quad \forall x \in \mathbb{R}^n$$

MATRIX TRANSFORMS FOLLOW THESE AXIOMS:

(a) $T(u+v) = T(u) + T(v) \quad \forall u, v \in \mathbb{R}^n$ (ADDITIVITY)

(b) $T(ku) = kT(u) \quad \forall u \in \mathbb{R}^n, k \in \mathbb{R}$ (HOMOGENEITY)

WE CAN FIND THE <u>STANDARD MATRIX</u> FOR A TRANSFORMATION $T: \mathbb{R}^n \to \mathbb{R}^m$ TO BE AN $m \times n$ MATRIX, GIVEN BY $A = [T(\hat{e}_1) | T(\hat{e}_2) | \cdots | T(\hat{e}_n)]$.

STEPS TO FIND A STANDARD MATRIX:

(1) FIND $T(\hat{e}_1), T(\hat{e}_2), \ldots, T(\hat{e}_n)$

(2) CONSTRUCT MATRIX $A = [T(\hat{e}_1) | T(\hat{e}_2) | \cdots | T(\hat{e}_n)]$.

eg/ CONSIDER $T: \mathbb{R}^n \to \mathbb{R}^m$ S.T. $T: (x_1, x_2) \longmapsto (3x_1 + x_2, 2x_1 - 4x_2)$.

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}3(1)+(0)\\2(1)-4(0)\end{bmatrix} = \begin{bmatrix}3\\2\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}3(0)+(1)\\2(0)-4(1)\end{bmatrix} = \begin{bmatrix}1\\-4\end{bmatrix}$$

$$\therefore A = [T(\hat{e}_1) | T(\hat{e}_2)] = \begin{bmatrix}3 & 1\\2 & -2\end{bmatrix}$$

A MAP $f: D \to R$ IS CALLED <u>LINEAR</u> IF:

(1) $f(x_1 + x_2) = f(x_1) + f(x_2)$ (ADDITIVITY)

(2) $f(kx) = kf(x)$ (HOMOGENEITY)

eg/ $f(x) = 3x$ IS LINEAR:

(1) $f(x_1 + x_2) = 3(x_1 + x_2) = 3x_1 + 3x_2 = f(x_1) + f(x_2)$

(2) $f(kx) = 3(kx) = k(3x) = kf(x)$

eg/ $f(x) = x^2$ IS NOT LINEAR

(1) $f(x_1 + x_2) = (x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2 = f(x_1) + f(x_2) \underline{\underline{+ 2x_1x_2}}$

WE DON'T NEED TO CHECK (2).