LAST TIME, WE TALKED ABOUT EUCLIDEAN VECTORS, WHICH YOU MIGHT HAVE BEEN FAMILIAR WITH FROM PHYSICS. BUT THIS REPRESENTS ONLY ONE TYPE OF VECTOR.

WHAT IS A VECTOR? IT'S ANYTHING IN A <u>VECTOR SPACE</u>.

BEGS THE QUESTION — WHAT IS A VECTOR SPACE?

A SET V IS CALLED A VECTOR SPACE IF ALL OF THESE ARE TRUE:

1) FOR ALL $u, v \in V$, $u+v \in V$ (CLOSURE UNDER ADDITION)
2) $u+v = v+u$ (COMMUTATIVITY OF ADDITION)
3) $u+(v+w) = (u+v)+w$ (ASSOCIATIVITY OF ADDITION)
4) THERE IS SOME VECTOR z SUCH THAT $u+z = z+u = u$ (ADDITIVE IDENTITY)
5) $u+(-u) = z$ (ADDITIVE INVERSE)
6) FOR ALL $k \in \mathbb{R}$, $u \in V$, $ku \in V$ (CLOSURE UNDER SCALAR MULT.)
7) $(k\ell)u = k(\ell u)$ (ASSOCIATIVITY OF SCALAR MULT.)
8) $1 \cdot u = u$ (MULTIPLICATIVE IDENTITY)
9) $k(u+v) = ku + kv$ (DISTRIBUTIVITY I)
10) $(k+\ell)u = ku + \ell u$ (DISTRIBUTIVITY II)

SOME EXAMPLES OF VECTOR SPACES:

- $\mathbb{R}^n$ (n-DIMENSIONAL VECTORS)
- P_2 (SET OF ALL QUADRATICS, LINES, AND CONSTANTS)
- $M_{2\times3}$ (SET OF 2×3 MATRICES)

NOTE: IF IT FAILS ANY ONE OF THE 10 AXIOMS, THEN IT'S NOT A VECTOR SPACE, NO NEED TO CONTINUE.

A <u>SUBSET</u> F OF A SET E MEANS THAT EVERYTHING IN F IS ALSO IN E, BUT NOT NECESSARILY VICE VERSA.

A <u>SUBSPACE</u> IS A SUBSET THAT IS ALSO A VECTOR SPACE.

THE 3-STEP SUBSPACE TEST:

1) CHECK THAT $V \neq \varnothing$ (ie, V IS NOT EMPTY)
2) CHECK CLOSURE UNDER ADDITION (AXIOM 1)
3) CHECK CLOSURE UNDER SCALAR MULT. (AXIOM 6)