TODAY, WE EXPLORE NEW WAYS TO CALCULATE DETERMINANTS

WE CAN FIND A DETERMINANT USING ROW REDUCTION:

eg/

$$\begin{bmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix} \xrightarrow[3]{R_1: R_1/3} \begin{bmatrix} 1 & -2 & 3 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix} \xrightarrow[1]{R_2: R_2+2R_1} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 3 & 4 \\ 0 & 1 & 5 \end{bmatrix} \xrightarrow[-1]{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 3 & 4 \end{bmatrix}$$

$$\xrightarrow[1]{R_3: R_3-3R_2} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & -11 \end{bmatrix} \xrightarrow[-11]{R_3: R_3/-11} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

so $\det \begin{bmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix} = 3 \cdot 1 \cdot (-1) \cdot 1 \cdot (-11) = 33$

NOW WE WILL INTRODUCE CRAMER'S RULE TO SOLVE DETERMINANTS.

eg/

$$\begin{cases} x - 4y + z = 6 \\ 4x - y + 2z = -1 \\ 2x + 2y - 3z = -20 \end{cases} \longrightarrow \begin{bmatrix} 1 & -4 & 1 \\ 4 & -1 & 2 \\ 2 & 2 & -3 \end{bmatrix} = A$$

$$\det(A) = \begin{vmatrix} 1 & -4 & 1 \\ 4 & -1 & 2 \\ 2 & 2 & -3 \end{vmatrix} = 3 - 16 + 8 + 2 - 4 - 48 = -55$$

NOW REPLACE COLUMN 1 WITH THE CONSTANTS COLUMN.

$$A_1 = \begin{bmatrix} 6 & -4 & 1 \\ -1 & -1 & 2 \\ -20 & 2 & -3 \end{bmatrix} \implies \det(A_1) = \begin{vmatrix} 6 & -4 & 1 \\ -1 & -1 & 2 \\ -20 & 2 & -3 \end{vmatrix} = 18 + 160 - 2 - 20 - 24 + 12 = 144$$

SAME FOR A_2 AND A_3 (REPLACING 2^{ND} AND 3^{RD} COLUMNS)

$\det(A_2) = \cdots = -89$

$\det(A_3) = \cdots = -230$

NOW WE HAVE A SOLUTION:

$$x_1 = \frac{\det(A)}{\det(A)} = \frac{144}{-55}, \quad x_2 = \frac{\det(A_2)}{\det(A)} = \frac{-89}{-55}, \quad x_3 = \frac{\det(A_3)}{\det(A)} = \frac{-230}{-55}$$

NO NEED FOR PAINFUL ROW REDUCTION!

A FEW BONUS POINTS:

$\det(AB) = \det(A)\det(B)$

$\det(kA) = k^n \det(A)$

$\det(A^T) = \det(A)$

$\det(A^{-1}) = \dfrac{1}{\det(A)}$