LAST TIME I MENTIONED THAT A SYSTEM IS CALLED <u>HOMOGENEOUS</u> IF ALL OF THE CONSTANT TERMS ARE 0. NOTICE THAT $x_1 = x_2 = \cdots = x_n = 0$ IS A SOLUTION ALWAYS. WE CALL THIS THE <u>TRIVIAL SOLUTION</u>. OTHER SOLUTIONS ARE CALLED <u>NON-TRIVIAL</u>.

A <u>SQUARE MATRIX</u> HAS THE SAME # OF COLUMNS AND ROWS. THAT # IS CALLED THE <u>ORDER</u> OF THAT MATRIX. THE ENTRIES WHOSE ROW AND COLUMN NUMBERS ARE THE SAME MAKE UP THE <u>MAIN DIAGONAL</u>.

WHEN ADDING MATRICES, WE ADD THEM ELEMENTWISE:

eg/ LET $A = \begin{bmatrix} 2 & 1 \\ 5 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

THEN $A+B = \begin{bmatrix} 4 & 1 \\ 5 & -3 \end{bmatrix}$

WE CAN SUBTRACT MATRICES SIMILARLY.

WE CAN ALSO MULTIPLY A MATRIX BY A SCALAR

eg/

$4A = \begin{bmatrix} 8 & 4 \\ 20 & -16 \end{bmatrix}$

TAKING THE <u>TRANSPOSE</u> OF A MATRIX IS LIKE "FLIPPING" IT, OR SWAPPING ITS ROWS AND COLUMNS.

eg/ $A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 4 & 2 \end{bmatrix} \longrightarrow A^T = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 1 & 2 \end{bmatrix}$

THE <u>TRACE</u> OF A SQUARE MATRIX IS THE SUM OF THE ENTRIES ON THE MAIN DIAGONAL.

eg/ $\mathrm{tr}\begin{bmatrix} 2 & 1 \\ 5 & -4 \end{bmatrix} = 2 - 4 = -2$

WE CAN ALSO MULTIPLY TWO MATRICES. (THIS WILL TAKE SOME GETTING USED TO)

eg/ LET $A = \begin{bmatrix} -3 & 6 \\ 1 & -2 \\ 7 & 4 \\ 1 & 3 \end{bmatrix}$ AND $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}$.

$AB = \begin{bmatrix} -3 & 6 \\ 1 & -2 \\ 7 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 9 & 0 & -9 \\ -3 & 0 & 3 \\ 15 & 18 & 21 \\ 7 & 5 & 3 \end{bmatrix}$

NOTE:
- $AB \neq BA$, SO ORDER MATTERS!
- YOU CAN ONLY MULTIPLY MATRICES IF THE # OF COLUMNS IN THE FIRST EQUALS THE # OF ROWS IN THE SECOND.
- IF THE FIRST MATRIX HAS n ROWS AND THE SECOND MATRIX HAS m COLUMNS THEN THE RESULTING MATRIX WILL BE $n \times m$.

A MATRIX INVERSE IS A MATRIX "A^{-1}" SO THAT $AA^{-1} = A^{-1}A = I$ WHERE I IS THE <u>IDENTITY MATRIX</u>, A SQUARE MATRIX WITH 1s ALONG MAIN DIAGONAL AND 0s EVERYWHERE ELSE.

NOTE: NOT ALL MATRICES ARE <u>INVERTIBLE</u>! IF A MATRIX IS NOT INVERTIBLE, WE CALL IT <u>SINGULAR</u>.

FORMULA FOR INVERSE OF A 2x2 MATRIX: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

EXTRA FACTS:
- $(A^{-1})^{-1} = A$
- $(A^T)^T = A$
- $(A \pm B)^T = A^T \pm B^T$
- $(A^T)^{-1} = (A^{-1})^T$
- $(kA)^{-1} = \frac{1}{k}A^{-1}$
- $(kA)^T = kA^T$
- $(AB)^T = B^T A^T$