LAST TIME, WE DISCUSSED EIGENVALUES AND EIGENVECTORS.

WE SAY THAT TWO MATRICES A AND B ARE "SIMILAR" IF WE CAN FIND AN INVERTIBLE MATRIX P SUCH THAT $A = P^{-1}BP$.

THIS IS "NICE" BECAUSE IF A AND B ARE SIMILAR, THEN THEY HAVE THE SAME DETERMINANT, INVERTIBILITY, RANK, NULLITY, TRACE, CHARACTERISTIC POLYNOMIAL, EIGENVALUES, AND EIGENSPACE DIMENSIONS.

SO, IF A IS A "HARD" MATRIX TO WORK WITH, BUT $B = P^{-1}AP$ AND B IS EASIER, WHY NOT JUST WORK WITH B INSTEAD?

THIS LEADS TO THE QUESTION — WHAT IS THE EASIEST TYPE OF MATRIX TO WORK WITH? THE DIAGONAL MATRIX.

WE SAY A MATRIX A IS <u>DIAGONALIZABLE</u> IF IT IS SIMILAR TO A DIAGONAL MATRIX.

TO FIND SUCH A MATRIX, WE NEED EIGENVECTORS TO BE L.I.

IF SO, FOLLOW THESE STEPS TO DIAGONALIZE A:

(1) CHECK TO MAKE SURE MATRIX IS DIAGONALIZABLE — SEE IF EIGENVECTORS ARE LINEARLY INDEPENDENT.

(2) IF IT IS DIAGONALIZABLE, THEN $P = [\xi^{(1)} \mid \xi^{(2)} \mid \cdots \mid \xi^{(n)}]$

(3) THEN $D = P^{-1}AP$, WHERE $D = \begin{bmatrix} \lambda_1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2 & 0 & \cdots & 0 \\ 0 & 0 & \lambda_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_n \end{bmatrix}$.

THIS LETS US EASILY COMPUTE POWERS OF A MATRIX!

$$D^k = \begin{bmatrix} \lambda_1^k & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2^k & 0 & \cdots & 0 \\ 0 & 0 & \lambda_3^k & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_n^k \end{bmatrix} \quad \text{SO} \quad A^k = PD^kP^{-1}$$

eg/ DIAGONALIZE $A = \begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix}$

$$A - I\lambda = \begin{bmatrix} -14-\lambda & 12 \\ -20 & 17-\lambda \end{bmatrix} \Rightarrow \det(A - I\lambda) = (-14-\lambda)(17-\lambda) + 240$$
$$\Rightarrow \det(A - I\lambda) = \lambda^2 - 3\lambda + 2 = (\lambda - 2)(\lambda - 1)$$

FOR $\lambda_1 = 1$:
$$(A - I\lambda)\xi^{(1)} = \begin{bmatrix} -15 & 12 \\ -20 & 16 \end{bmatrix}\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \xi^{(1)} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

FOR $\lambda_2 = 2$:
$$(A - I\lambda_2)\xi^{(2)} = \begin{bmatrix} -16 & 12 \\ -20 & 15 \end{bmatrix}\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \xi^{(2)} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

CHECK THAT $\{\xi^{(1)}, \xi^{(2)}\}$ IS L.I. (IT IS!)

SO $P = \begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix}$ AND $P^{-1} = \frac{1}{16-15}\begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$

CHECK $P^{-1}AP = D$ WHERE $D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

THEN WHAT IS A^{10}? WELL $D^{10} = \begin{bmatrix} 1 & 0 \\ 0 & 1024 \end{bmatrix}$

SO MULTIPLY $PD^{10}P^{-1}$ TO GET A^{10}! THATS 2 MULTIPLICATIONS INSTEAD OF 10.