

Practice Problems

MATH2055: Advanced Linear Algebra Tutorial 6 **Midterm Review**

Benjamin Fedoruk

Ontario Tech University

April 4, 2024

Question 1 - Abstract Vector Spaces

Let β be any positive real number. Show that \mathbb{R} is a real vector space, using addition \oplus and scalar multiplication \odot .

$$x \oplus y = \beta xy$$

$$k \odot x = \beta^{k-1} x^k$$

Question 2 - Span of Three 2×2 Matrices

What is $\text{span}\{A, B, C\}$?

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Question 3 - Linear or Not

Is the following transformation linear?

$$T : \mathbb{R}^{\mathbb{R}} \rightarrow \mathbb{R}^2 \quad T(f) = \begin{bmatrix} f(0) \\ f(1) + 1 \end{bmatrix}$$

Question 4 - Pythagorean Theorem

Prove the Pythagorean Theorem for inner product spaces:
Let v and u be orthogonal vectors. Then

$$\|v + u\|^2 = \|u\|^2 + \|v\|^2$$