GENERAL TECHNIQUES FOR PROOF:

1) DIRECT PROOF
2) CONTRAPOSITIVE

eg/ WHEN IT RAINS, THE SIDEWALK GETS WET.
THE SIDEWALK IS CURRENTLY DRY. WHAT DOES THIS TELL US?
WHAT IF THE SIDEWALK IS WET? (THIS IS AN EXAMPLE OF "CONVERSE")

3) CONTRADICTION

THIS IS PROBABLY THE MOST DIFFICULT TECHNIQUE AS IT CAN TAKE MANY FORMS. BUT VERY INTUITIVE.

4) WEAK INDUCTION

eg/ HOW CAN WE PROVE THAT, GIVEN A SETUP OF DOMINOES, ALL DOMINOES WILL FALL?

IT ALWAYS HELPS TO WRITE OUT YOUR ASSUMPTIONS AND WHAT YOU WANT TO SHOW. THIS MAY HELP GIVE YOU A "STROKE OF GENIUS".

TIPS FOR WHEN YOU'RE STUCK:

- GO FOR A NICE WALK (AT LEAST 1 HR)
- CALL YOUR MOM/DAD/SIBLING/FRIEND
- GO FOR COFFEE

REMEMBER: UNDERGRADS ARE STATISTICALLY THE MOST LIKELY TO MAKE MOVEMENT ON AN OPEN PROBLEM!

VECTOR SPACE AXIOMS:

- COMMUTATIVITY — $u+v=v+u \ \forall u,v\in V$
- ASSOCIATIVITY — $(u+v)+w=u+(v+w) \ \forall u,v,w\in V$
- ADDITIVE IDENTITY — $\exists z\in V$ S.T. $z+v=v+z=v \ \forall v\in V$
- ADDITIVE INVERSE — $\forall v\in V \ \exists w\in V$ S.T. $v+w=0$
- MULTIPLICATIVE IDENTITY — $1v=v \ \forall v\in V$
- DISTRIBUTIVITY — $a(u+v)=au+av$ AND $(a+b)v=av+bv \ \forall a,b\in\mathbb{F},\ v,u\in V$.

WAIT, WHAT'S THAT $\mathbb{F}$? THIS IS CALLED A "FIELD". IT'S BASICALLY A SET WHICH HAS ADDITION, SUBTRACTION, MULTIPLICATION & DIVISION.

SYMBOL A^B MEANS $\{f \mid f: B\to A\}$.

A SUBSPACE IS JUST A SUBSET OF A VECTOR SPACE WHICH IS, ITSELF, A VECTOR SPACE.

STRATEGY: INSTEAD OF PROVING SOME SET W IS A VECTOR SPACE, JUST FIND SOME "BIGGER" VECTOR SPACE V S.T. $W\subseteq V$, THEN USE THE 2-STEP SUBSPACE TEST.

2-STEP SUBSPACE TEST:

1) CHECK $u+v\in W$ IF $u,v\in W$
2) CHECK $ku\in W$ IF $u\in W$, $k\in\mathbb{F}$.